A Robust Control Approach to Formation Control

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***Abstract*— This paper considers formation control for multi-agent systems (vehicles, robots, satellites, etc.) and is based on the work of Fax and Murray [1], which provides a link between graph theory and the formation control problem for a given communication topology. We propose a distributed controller H_∞ and μ synthesis techniques that can guarantee the stability of the multi-agent system for any number of agents and any communication topology. Two examples illustrate the proposed method.**

I. Introduction

A topic of considerable interest to the control community over the last 10 years is control of multi-agent systems. Such systems comprise two or more physically independent agents/units (mobile robots, autonomous vehicles, etc.) that have a common goal and can communicate with each other.

In this paper we consider the control problem for formations of N identical agents, communicating with each other (for an overview of the work in multi-agent consensus, refer to [2]). As shown in [1], the communication topology can be represented as a directed-graph, where the nodes are the agents and the vertices are the communication links - see Fig. 1. An arrow from agent k to agent i shows an information flow, i.e. unit i receives information (senses, sees, etc.) from agent k (in [1] the arrows are in opposite directions). In the example in Fig. 1 agent 5 receives information (only) from unit 3 and sends information to agent 3 and 4. Since agent 1 does not receive any external information, it acts as a leader for the formation.

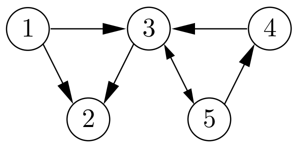

Fig. 1. Graph representation of a topology with $N = 5$ units

Such decentralized multi-agent systems offer, among others, the following advantages over centralized systems:

- failure of one agent does not mean failure of the whole system;
- the system can be flexibly adjusted to other tasks by changing the number/type of agents;
- less communication and computation overhead as compared to centralized control of the agents;
- cheaper manufacturing the replacement of an agent.

In this paper we will use the term 'unit' to denote a single element of the system, since 'agent' is mainly associated with higher level planning and decision capabilities than the ones considered here.

In [1], [3], Fax and Murray consider the problem of controlling a formation of identical units and establish a link between graph-theory and the communication topology within the formation. Furthermore they derive formation stability tests, based on the dynamics of a single unit and its controller.

This framework is used later in [4], [5], [6] for distributed Linear Quadratic Regulator (LQR) design for formations. An output-feedback approach based on a decomposition approach and linear matrix inequalities is proposed in [7].

Whereas all of the above methods consider a fixed topology, in practice the topology might change due to obstacles, environmental disturbances, failure of units, or the desire to dynamically adapt the number of units in the formation to new tasks or environmental changes. In this paper, we use the framework of [1] and employ results from graph-theory to convert the formation stability problem into a robust controller-design problem for a single unit. A controller satisfying the design requirements will be shown to guarantee that all possible topologies (for formations with any number of units N and any communication between the units) will be stable. We will also show that the technique can easily handle performance requirements.

Furthermore, whereas previous design methods are applicable only to undirected communication topologies (or require special attention for directed ones [7]) the proposed technique is designed to handle directed communication and include the undirected one as a special case. Such communication is of practical importance when units use cameras, proximity sensors, etc. to obtain information.

The paper is structured as follows. In Section II the graph-representation of formation, units' and controllers' dynamics as well as conditions for formation stability are briefly introduced. Section III presents the robust-stability and robust-performance design formulations for controller design for all formations. The design method is illustrated by two design examples in Section IV. Finally, conclusions are drawn in Section V.

The following notation will be used throughout the paper:

I_q denotes the $q \times q$ identity matrix; $\mathbf{R}^{p\times q}$, $\mathbf{C}^{p\times q}$ are, correspondingly, the sets of $p \times q$ real and complex matrices; j is the complex unit; $\text{lft}(P(s), K(s))$ denotes the lower linear fractional transformation of $P(s)$ with $K(s)$ [8]; $\otimes$ is the Kronecker product.

II. Formation Stability

A. Local Dynamics and Stability

Assume that each unit (without its controller) is described by the same linear time-invariant (LTI) model $P(s)$. A state-space realization of the i-th unit is

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i \\ y_i &= C_f x_i + D_f u_i \\ \phi_i &= C_l x_i + D_l u_i \end{aligned} \tag{1}$$

where the indices l and f denote, respectively, local-level and formation-level matrices. Here $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times h}$, $C_l \in \mathbf{R}^{m\times n}$ and $D_l \in \mathbf{R}^{m\times h}$ are constant and known, n, h and m are respectively the number of states, controlled inputs and measured outputs of each unit. Furthermore, u_i is the control input and ϕ_i denotes the measured outputs. These outputs are used for a local feedback control that ensures local stability of the unit.

The additional p outputs y_i, represent outputs that are of interest on *formation-level*, where $C_f \in \mathbf{R}^{p\times n}$ and $D_f \in \mathbf{R}^{p\times h}$. Considering for example a walking robot, the local signals of interest could be joint positions etc., whereas on formation-level the signals of interest might be only the x and y coordinates of the robot in the operating space (for some applications $C_f = C_l$, $D_f = D_l$). In the following, without loss of generality we consider $D_l = 0$, $D_f = 0$.

Let all units be controlled locally by identical LTI controllers $K(s)$, as shown in Fig. 2, where a state-space model of the i-th controller is

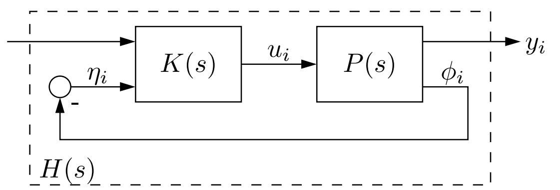

Fig. 2. Local feedback of single unit and its controller

$$\begin{aligned} \dot{v}_i &= \Phi v_i + \Psi_f e_i + \Psi_l \eta_i \\ u_i &= \Gamma v_i + \Upsilon_f e_i + \Upsilon_l \eta_i \end{aligned} \tag{2}$$

Here $\eta_i = -\phi_i$ is the local control error and e_i is the formation-level control error, which will be introduced later.

The local stability of an unit is then equivalent to the stability of

$$H(s) = \text{lft}\,(P(s)K(s),\, -I_m) \tag{3}$$

B. Formation Stability

The formation-level control error e_i, as defined in [1], is the average of errors in the difference of outputs between unit i and the elements of the set $\mathcal{J}_i$ of units from which unit i receives information (e.g. in Fig. 1, $\mathcal{J}_1 = \{\}$, $\mathcal{J}_2 = \{1, 3\}$, etc.).

$$e_i = \frac{1}{|\mathcal{J}_i|} \sum_{k\in\mathcal{J}_i} e_{ik} \tag{4}$$

Here $|\mathcal{J}_i|$ is the size of the set, i.e. the number of units from which unit i receives information. The term e_{ik} is the error between the i-th and k-th unit

$$e_{ik} = \bar{r}_{ik} - (y_i - y_k)$$

where $\bar{r}_{ik}$ is a prescribed difference between the outputs. For a topology where unit i is a leader, we define $e_i = r_i - y_i$, thus allowing a direct reference input to the leader.

Equation (4) represents a practical way of defining the formation-level control error, since each unit relies only on information from the neighbors it can sense and has to determine its action depending on it. It is possible, although not done here, to assign different weights on the errors e_{ik} depending on their importance or the reliability of the provided information from unit k.

The topology of a formation can be described by the (normalized) Laplacian matrix $L \in \mathbf{R}^{N\times N}$, defined as

$$L_{ik} = \begin{cases} 1, & \text{if } i = k \\ -\frac{1}{|\mathcal{J}_i|}, & k \in \mathcal{J}_i \\ 0, & k \notin \mathcal{J}_i \end{cases} \tag{5}$$

From the definition of the Laplacian, it follows that each row adds up to zero (except for formations with leader), thus 0 is an eigenvalue of L and L is not invertible. The Laplacian matrix of the formation in Fig. 1 is given in Appendix A. The results presented below will hold also if different weights are assigned to the communication channels, as long as the row sum of L is zero.

Let $L_{(p)} = L \otimes I_p$, $e = \begin{bmatrix} e_1^T & \dots & e_N^T \end{bmatrix}^T$ and $\hat{y} = \begin{bmatrix} y_1^T & \dots & y_N^T \end{bmatrix}^T$. Then from (4) follows

$$e = \bar{r} - L_{(p)}\hat{y} = L_{(p)}(\hat{r} - \hat{y})$$

The $\hat{r}$ reference signals define a desired value for the units' outputs (absolute reference), i.e. they prescribe both the shape and the position of the formation, whereas $\bar{r}$ defines a reference for the relative position of the units in the formation (relative reference), that is only the formation's 'shape'. Note that since L is not invertible, to the same relative references correspond infinitely many absolute references.

The closed-loop interconnection of the whole formation is shown in Fig. 3.

The following Theorem is taken from [1].

Theorem 1 *A local controller $K(s)$ stabilizes the closed-loop formation with a topology described by the Laplacian*

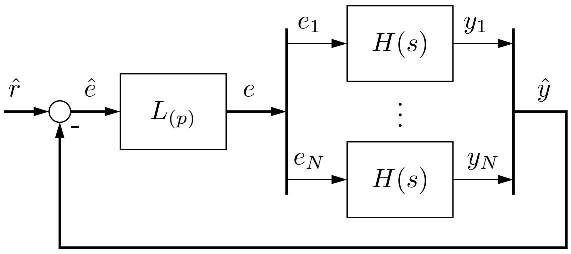

Fig. 3. Closed-loop representation of a formation

matrix L if and only if it simultaneously stabilizes the set of systems

$$\begin{aligned} \dot{\tilde{x}}_i &= A\tilde{x}_i + Bu \\ y &= \lambda_i C_f \tilde{x}_i \\ \phi &= C_l \tilde{x}_i \end{aligned} \tag{6}$$

where λ_i are the eigenvalues of L; $i = 1, \ldots, N$.

C. Eigenvalues of the Laplacian

Finally, we recall some results from graph theory (see, e.g., [1], [9], [10]).

Lemma 1 *Every eigenvalue λ of the (normalized) Laplacian matrix L belongs to the closed disk with radius 1 centered at $1 + j0$, that is* $|\lambda - 1| \leq 1$.

This disk is called the *Perron disk*, see Fig. 4.

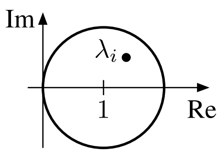

Fig. 4. Eigenvalues of the graph-Laplacian

The eigenvalues λ can be equivalently represented as

$$\lambda = \{1 + \delta_\lambda \mid \delta_\lambda \in \mathbf{C},\ |\delta_\lambda| \leq 1\} \tag{7}$$

For undirected graphs the Laplacian matrix is symmetric and thus has only real eigenvalues, i.e. $0 < \lambda_i < 2$ for $i = 1, \ldots, N$.

III. Stabilizing All Topologies

Theorem 1 is used in [1] to derive a Nyquist-test like stability criterion for a given communication topology in the case of SISO units; a possible extension to the MIMO case is briefly mentioned. Here we consider the MIMO case and in contrast to [1] consider not a particular communication topology, but all possible ones. Further, we add performance requirements.

A. Stability Conditions

Substitute λ from (7) in the equation for y in (6) to obtain

$$\begin{aligned} \dot{\tilde{x}}_i &= A\tilde{x}_i + Bu \\ y &= C_f \tilde{x} + \delta_\lambda C_f \tilde{x} \\ \phi &= C_l \tilde{x}_i \end{aligned}$$

By augmenting the plant with q additional 'uncertainty' inputs w and outputs z we obtain the system $G(s)$

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + Bu \\ z &= C_\delta \tilde{x} \\ y &= C_f \tilde{x} + D_\delta w \\ \phi &= C_l \tilde{x} \end{aligned} \tag{8}$$

where

$$w = \delta_\lambda I_q z$$

and $D_\delta C_\delta = C_f$; $D_\delta \in \mathbf{R}^{p\times q}$; $C_\delta \in \mathbf{R}^{q\times n}$. Such D_δ and C_δ matrices can be obtained either by selecting one of them equal to C_f and the other as identity matrix, or by factoring C_f (e.g., via singular-value decomposition (SVD)).

We can now prove the following Theorem.

Theorem 2 *Consider the closed-loop formation in Fig. 3. A controller $K(s)$ as in* (2) *stabilizes the formation with every fixed topology if it stabilizes* (8) *for all* $|\delta_\lambda| \leq 1$.

Proof: For every δ_λ equation (8) can be transformed to (6). Thus a controller guaranteeing the stability of (8) for every $|\delta_\lambda| \leq 1$ will guarantee the stability of (6) for all λ inside the Perron disk. Since the eigenvalues of all topologies lie inside the Perron disk (Lemma 1), and using Theorem 1, it follows that the controller will guarantee the stability of the formation under any topology. ■

The above theorem provides a sufficient condition for the stability of a formation with any number of units N and any communication topology - one needs to check the robust stability of (8) under all $|\delta_\lambda| \leq 1$. The last can be easily checked with tools from robust control theory [8].

Remark 1 The condition in Theorem (2) is conservative for topologies with a finite number of units, since the eigenvalues of L can take only a finite number of values. For example, the eigenvalues of the Laplacian matrices for all possible communication topologies with $N = 4$ units, are shown in Fig. 5.

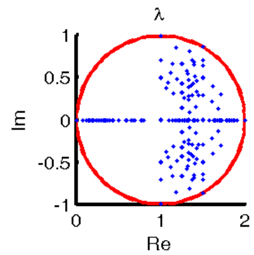

Fig. 5. Eigenvalues of all possible Laplacian matrices of topologies with 4 units

B. Controller Synthesis

The condition in Theorem 2 can now be extended to controller synthesis. Let $\Delta \in \mathbf{\Delta}$ be a norm bounded uncertainty acting on the generalized plant (8), i.e. $w = \Delta z$, where

$$\mathbf{\Delta} := \{\Delta \mid \Delta \in \mathbf{C}^{q\times q}, \|\Delta\| \le 1\}$$

Let $T(s) = \text{lft}(G(s), -K(s))$ (see Fig. 6) be the closed-loop transfer function from w to z, where $G(s)$ is given by (8), and $\epsilon = \begin{bmatrix}-y^T & -\phi^T\end{bmatrix}^T$.

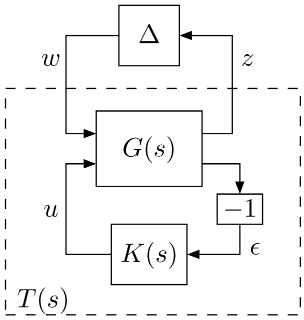

Fig. 6. Closed-loop interconnection for controller design

Theorem 3 *Consider the closed-loop formation in Fig. 3. A controller $K(s)$ as in* (2) *stabilizes the formation under every time-varying topology if it satisfies $\|T(s)\|_\infty < 1$.*

Proof: The stability of the formation with every topology follows from the construction of the generalized plant $G(s)$ and Theorem 2. The stability under time-varying topology (e.g., change of the number of units, establishing or losing communication links) follows from the fact that H_∞ synthesis with full-block uncertainty Δ is equivalent to quadratic stability under time-varying uncertainty (see, e.g., [11]). ■

The above theorem does not only provide a method for analyzing whether a controller stabilizes the formation under every fixed/time-varying topology, but also a means of synthesizing such controllers using standard H_∞ tools.

Remark 2 When using the above theorem for synthesis, the distributed controller design problem is reduced to the H_∞-synthesis problem for a single unit. This problem can be efficiently solved and presents a considerable complexity reduction for large formations ($N >> 1$) compared to previous methods [4], [5], [6], [7]. Furthermore, if a controller satisfying $\|T(s)\|_\infty < 1$ exists it will guarantee stability under every fixed topology and, moreover, under every time-varying topology.

Theorem 3 is however a conservative condition for stability under every topology, since the H_∞-analysis/synthesis considers the uncertainty Δ (see Fig. 6) to be a full-block complex one, whereas for formations it is a diagonal matrix $\delta_\lambda I_q$. This conservatism can be reduced by using the structured singular value μ [12], [13].

Theorem 4 *Consider the closed-loop formation in Fig. 3. Let $\mathbf{\Delta} := \{\delta_\lambda I_q \mid |\delta_\lambda| \le 1\}$. A controller $K(s)$ as in* (2) *stabilizes the formation under every fixed topology if it satisfies $\mu_{\mathbf{\Delta}}(T(s)) < 1$.*

Proof: Follows directly from Theorem 2 and the definition of the structured singular value. ■

Note that, whereas Theorem 3 guarantees the stability under time-varying topologies, Theorem 4 guarantees it only for fixed ones.

Remark 3 For formations with symmetric communication structure, the eigenvalues of the Laplacian are real and the problem can be solved as real-valued μ-synthesis to further reduce the above conservatism.

In addition to the stability requirements, one can add performance requirements (e.g. using the mixed-sensitivity framework) or/and add additional inputs and outputs connected to uncertainty block(s) (which includes model uncertainty, or relaxes the requirement for identical units). The closed-loop interconnection with added performance channels w_P and z_P is shown in Fig. 7, where now $\tilde{G}(s)$ is a generalized plant including $G(s)$ and shaping filters. Note that this imposes performance requirements on a single unit rather than on the whole formation, and can thus be viewed as worst-case performance requirement on a unit.

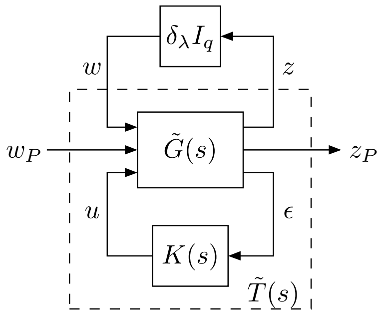

Fig. 7. Closed-loop interconnection for robust-performance controller design

The synthesis problem is then characterized by the structured uncertainty

$$\mathbf{\Delta} := \left\{ \begin{bmatrix} \delta_\lambda I_q & 0 \\ 0 & \Delta_P \end{bmatrix} \;\middle|\; \begin{array}{l} \delta_\lambda \in \mathbf{C},\ \|\delta_\lambda\| \le 1 \\ \Delta_P \in \mathbf{C}^{v\times d},\ \|\Delta_P\| \le 1 \end{array} \right\}$$

where Δ_P is a fictitious uncertainty, corresponding to the performance channels: $w_P = \Delta_P z_P$. Such structured uncertainty can be handled again using the structured singular value theory.

IV. Design Examples

In this section we illustrate the proposed design technique via a simple SISO example. The system we consider is the single-dimensional dynamics of a hovercraft [1], [3], [2]

$$\bar{P}(s) = \frac{1}{s^2}e^{-sT_d}$$

It is assumed that the hovercraft can move independently in x and y direction, and has the same dynamics in both directions, in which case the problem can be reduced to a single-dimensional one. Once a controller for one axis it is obtained, it can be applied to both axes. Let $T_d = 0.1$ sec. In the following a 2nd order Padé approximation will be used for the time delay, and $P(s)$ will denote the approximated system.

A. Formation-Level Controller Design

We first make the same assumption as in [1], [3], that the local feedback-loop is already closed. In this case the controller $K(s)$ has state-space realization

$$\begin{aligned} \dot{v}_i &= \Phi v_i + \Psi_f e_i \\ u_i &= \Gamma v_i + \Upsilon_f e_i \end{aligned} \tag{9}$$

The open-loop transfer function is $H(s) = P(s)K(s)$. The output matrix C_f is factorized via SVD, which results in

$$C_\delta = \begin{bmatrix} 0 & 0.0143 & -0.2144 & 1.0719 \end{bmatrix}, \quad D_\delta = 1.0932$$

Since in this case $q = 1$, Theorems 3 and 4 will provide the same result. Because this system is SISO one can verify the results of the controller design by the Nyquist stability test for formations proposed in [1]. The test states that a formation is stable if the Nyquist diagram of $H(s)$ does not encircle any of the points $-1/\lambda$. Since the Perron disk is mapped to the vertical line passing through -0.5, the stability of every fixed formation will be guaranteed if the Nyquist diagram lies right of the -0.5 line for all frequencies.

The controller design returns a 4th order controller achieving $\|T_{zw}(s)\|_\infty = 1.004$. The Nyquist diagram of $H(s)$ is shown in Fig. 8. From both the norm and the Nyquist diagram it is obvious that the controller does not stabilize every possible formation. To understand why, recall that $\lambda = 0$ is always an eigenvalue of L. Substituting it in (8) and taking in account that $P(s)$ is the system with inner-loop already closed, it follows that a formation-level controller can stabilize every formation only if the unit dynamics are open-loop stable. Since $P(s)$ contains a double integrator this is not the case here.

B. Local- and Formation-Level Controller Design

To overcome the above problem and illustrate the simultaneous local-level and formation-level controller design, let us now assume that $P(s)$ contains only the uncontrolled unit's dynamics and that the same measurements are used for both formation-level and local-level control, i.e., $K(s)$ as in (2) and $\phi_i = y_i$.

We use the sensitivity weighting filter $W_S(s)$ to express requirements on the reference tracking and the speed of response. Note that either $\hat{e}$ or e can be used as input to $W_S(s)$ (see Fig. 3). In the former case this will account for the absolute error of the formation, which can be arbitrarily large in the absence of a leader (e.g., by moving the whole formation). Therefore here we will use the latter error signal.

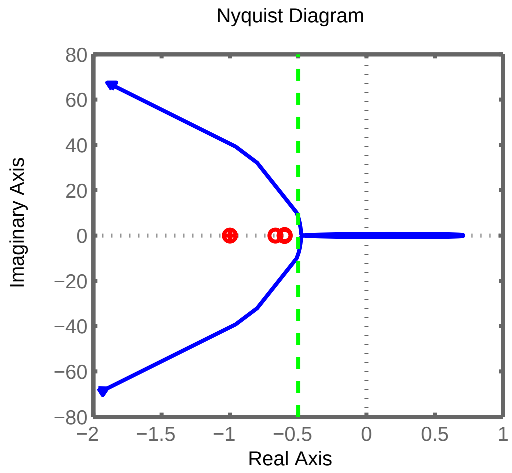

Fig. 8. Nyquist plot of $H(s)$ and the negative inverse eigenvalue of the Laplacian corresponding to the graph in Fig. 1

The generalized plant is shown in Fig. 9, where $w_\delta = \delta_\lambda I_q z$ and $\hat{G}(s)$ corresponds to the state-space model

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + Bu \\ z &= -C\tilde{x} + I_p r \\ e &= -C\tilde{x} + I_p w + I_p r \\ \eta &= -C\tilde{x} \end{aligned} \tag{10}$$

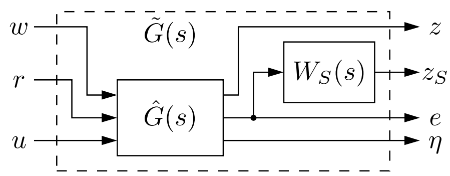

Fig. 9. Generalized plant for simultaneous local-level and formation-level robust-performance controller design

Using $W_S(s) = \frac{2}{s+0.02}$ and H_∞-synthesis, a 5th order controller was obtained that achieves a closed-loop H_∞ norm of 1.2293. However, since the H_∞ norm from w to z is 0.9937, this controller is guaranteed to stabilize the formation under every fixed/time-varying topology. As can be seen from the Nyquist plot of $H(s)$ shown in Fig. 10, the Nyquist curve stays right from the -0.5 line for all frequencies, thus confirming that the obtained controller stabilizes every formation.

To illustrate that the formation remains stable also under change in the communication topology, we simulate the formation response. For visualization purposes we consider only the one-dimensional unit's dynamics along the y-axis. The formation consists of $N = 5$ units with communication topology as in Fig. 1. All the units start from position 0 and are given reference distance of 1, i.e. unit one should end at position 1, unit 2 at position 2, etc. Five seconds after the simulation start we switch the topology to a cyclic one, i.e. unit one receives information only from unit 2, unit 2 only

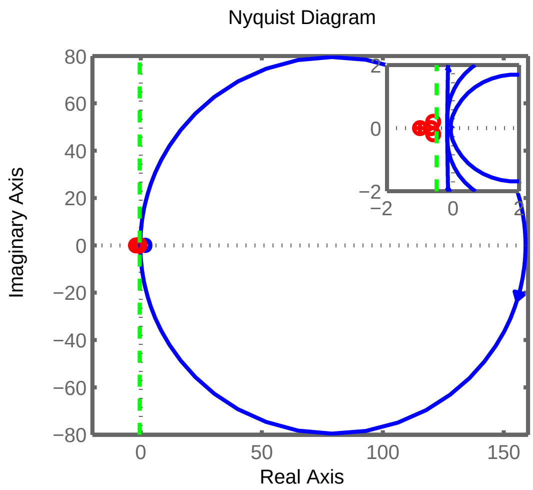

Fig. 10. Nyquist plot of $H(s)$ by simultaneous local-level and formation-level controller design and a zoomed-in view of the origin.

from unit 3, ..., unit N from unit 1. The response of the formation is shown in Fig. 11, where the upper plot shows the units' outputs and the lower one the error signals. The (green) dashed line shows the response when no topology change occurred.

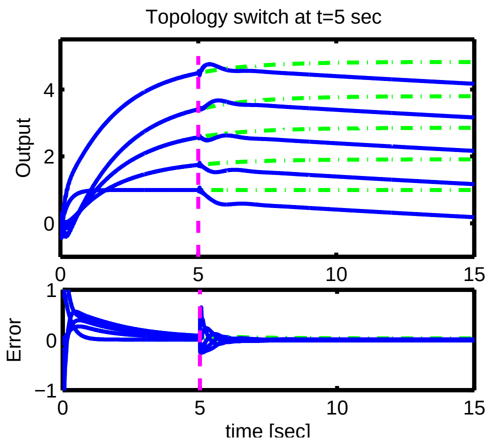

Fig. 11. Formation response to a position reference and topology change. Top figure - unit's outputs, bottom figure - unit's error signals.

Whereas for the formation in Fig. 1 unit 1 is a leader, the cyclic formation has no leader, which explains why the formation does appear to drift. However the formation does take the desired shape, as seen from the error plot (and in fact converges to a fixed position after approximately 150 sec).

V. Conclusions and Future Work

In this paper we presented a sufficient condition for a controller to stabilize a formation under every fixed communication topology and any number of units. We then showed how the problem can be casted in the robust-stability H_∞ controller design framework, and showed that this approach offers the following advantages.

- A controller satisfying the design requirements will guarantee the formation stability not only for every fixed topology, bus also for time-varying topologies.
- The distributed controller design problem for a formation is reduced to the H_∞-synthesis problem for a single unit.

We have further shown that μ-synthesis can be used to relax the conservatism of the H_∞ design but at the price of not guaranteeing stability for time-varying topologies.

Finally we illustrated robust-stability formation-level controller design and simultaneous formation-level and local-level designs via examples.

The directions of future research include obtaining necessary and sufficient conditions for the formation stability under every topology, and obtaining a tighter uncertainty description for fixed topologies.

Appendix

A. Laplacian Matrix

The Laplacian of the graph in Fig. 1 is

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{3} & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

The eigenvalues λ of L are 0.1486, $1.4257 \pm j0.4586$, 1, 1.

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